12.2.18

$P(A\cap B) = P(A) + P(B) - P(A\cup B) = 0.4 + 0.4 - 0.7 = 0.1$

$P(A^c \cap B^c) = P((A\cup B)^c) = 1 - P(A\cup B) = 0.3$

12.2.34

Let $A = \{$at least 1 child is boy$\}$, we have $A^c = \{$all children are girls$\}$

Since each child is independent to the others, we have

$P(A^c) = (\frac{1}{2})^3 = \frac{1}{8}$

Thus $P(A) = 1 - P(A^c) = \frac{7}{8}$

12.2.40

Let $B = \{$at least 2 out of 3 balls are green$\}$

$B_1 = \{$exactly 2 balls are green$\}$

$B_2 = \{$exactly 3 balls are green$\}$

we have $B = B_1 \cup B_2$ and $B_1 \cap B_2 = \phi$

Thus $P(B) = P(B_1) + P(B_2) = \dfrac{\binom{3}{2}\binom{5}{1}}{\binom{8}{3}} + \dfrac{\binom{3}{3}}{\binom{8}{3}}$

12.3.25

Let $H = \{\text{outcome is head}\}$

$B_1 = \{\text{the coin you pick is fair}\}$

$B_2 = \{ \cdots\cdots \text{ is unfair}\}$

Note that $P(H|B_1) = \frac{1}{2}$, $P(H|B_2) = 1$

So

$$P(H) = P(H|B_1)P(B_1) + P(H|B_2)P(B_2)$$

$$= \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}$$

$$= \frac{3}{4}$$

12.3.39

From the previous problem, we know $P(H) = \frac{3}{4}$

$$P(B_1|H) = P(B_1 \cap H) / P(H)$$

$$= \frac{P(H|B_1)\,P(B_1)}{P(H)}$$

$$= \frac{1}{4} \Big/ \frac{3}{4} = \frac{1}{3}$$